

# Application of Biclustering to the Discovery of Constant and Gradual Patterns



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## 1. Introduction

Biclustering is a **simultaneous grouping** of rows and columns of a matrix. Here we are focusing on **constant-column** and **coherent-sign-changes** biclustering. We show that those types of biclustering can be used in the problem of **gradual pattern mining**.

## 2. Biclustering

### Difference between clustering and biclustering

Clustering					
	A	B	C	D	E
1					
2					
3					
4					
5					

Grouping of rows

Biclustering					
	A	B	C	D	E
1					
2					
3					
4					
5					

Grouping of rows and columns

### Some types of bicluster

#### Constant-value bicluster

2	2	2	2
2	2	2	2
2	2	2	2
2	2	2	2

#### Constant-column (CC) bicluster

2	5	1	6
2	5	1	6
2	5	1	6
2	5	1	6

#### Coherent-sign-changes (CSC) bicluster

+	+	-	+
+	+	-	+
-	-	+	-
+	+	-	+

All rows jointly are in a given “agreement” with the columns

### Relation of CC and CSC biclusters

	a	b	c	d	e
1	-	-	-	-	+
2	-	-	-	-	+
3	-	-	-	+	+
4	+	+	-	+	+
5	-	+	+	-	+
6	+	-	+	-	-

CSC bicluster {4, 5}, {a, c, d}  
 =  
 CC bicluster {4, 5}, {ac, ad}

	ab	ac	ad	ae	bc	bd	be	cd	ce	de
1	S	S	S	D	S	S	D	S	D	D
2	S	S	S	D	S	S	D	S	D	D
3	S	S	D	D	S	D	D	D	D	S
4	S	D	S	S	D	S	S	D	D	S
5	D	D	S	D	S	D	S	D	S	D
6	D	S	D	D	S	D	D	S	S	D

S = same  
 D = different

## 3. Partition Pattern Structures for CC Biclustering

### Partition (of object)

	$m_1$	$m_2$	$m_3$	$m_4$	$m_5$
$g_1$	1	5	3	2	7
$g_2$	1	1	4	2	7
$g_3$	2	5	4	5	3
$g_4$	2	5	4	5	7

A partition  $d = \{p_i\}$  of a set of objects  $G$  is a collection of  $p_i \subseteq G$  such that there is no overlap among  $p_i$ .

Given  $G$  as the set of objects and  $M$  as the set of attributes, then a partition function  $\delta: M \rightarrow D$  maps an attribute  $m$  to a partition  $d$ .

Example:

$$\delta(m_1) = \{\{g_1, g_2\}, \{g_3, g_4\}\}$$

### Similarity and order between two partitions

$$d_1 = \{p_i\} \text{ and } d_2 = \{p_j\}$$

$$d_1 \sqcap d_2 = \bigcup_{i,j} \{p_i \cap p_j\}$$

$$d_1 \sqsubseteq d_2 \Leftrightarrow d_1 \sqcap d_2 = d_1$$

Example:

$$\delta(m_1) = \{\{g_1, g_2\}, \{g_3, g_4\}\}$$

$$\delta(m_2) = \{\{g_1, g_3, g_4\}, \{g_2\}\}$$

$$\delta(m_1) \sqcap \delta(m_2) = \{\{g_1\}, \{g_2\}, \{g_3, g_4\}\}$$

### Partition pattern concept and CC bicluster

Given a set of attribute  $M$  and partition space  $D$ , a pair  $(A, d)$  is a partition pattern concept iff  $A' = d$  and  $d' = A$ , where:

$$A' = \bigcap_{m \in A} \delta(m)$$

$$A \subseteq M$$

$$d' = \{m \in M \mid d \sqsubseteq \delta(m)\}$$

$$d \in D$$

Example:

A concept  $(\{m_1, m_4\}, \{\{g_1, g_2\}, \{g_3, g_4\}\})$

Two CC biclusters:  $(\{g_1, g_2\}, \{m_1, m_4\})$  and  $(\{g_3, g_4\}, \{m_1, m_4\})$

## 4. Gradual Patterns

Hotel	City population	Distance from center	Price
$h_1$	1000	5	45
$h_2$	500	10	25
$h_3$	600	1	50
$h_4$	40000	2	100

The hotels in a **big cities** are **more expensive** than those in a **smaller cities**.  
 The hotels **near city center** are **more expensive** than those **far from center**.

Attribute	$h_1 h_2$	$h_1 h_3$	$h_1 h_4$	$h_2 h_3$	$h_2 h_4$	$h_3 h_4$
Pop.	>	>	<	<	<	<
Dist.	<	>	>	>	>	<
Price	>	<	<	<	<	<

CSC bicluster

Four pairs of hotels (out of six) show the relation among city population, hotel's distance from city center, and hotel's price.

## 5. References

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